SEMESTER-1 (Period-II)

P11CH2

TOPIC

# 2 Composition and Resolution of Force

## 2.1. PROPERTIES OF VECTOR ADDITION

*A quantity can be a vector only if it obeys the laws of vector addition.*

Following are the important properties of vector addition.

**(*i*) Vectors of the same nature alone can be added.** A force vector can be added to force vector only. It cannot be added to displacement vector.

**(*ii*) Vector addition is commutative.** *The sum of the vectors remains the same in whatever order they may be added.*

According to commutative law of vector addition,

$$\vec{a} + \vec{b} + \vec{c} + \ldots\ldots = \vec{b} + \vec{a} + \vec{c} + \ldots\ldots = \vec{c} + \vec{a} + \vec{b} + \ldots\ldots$$

The result of vector addition does not depend on the order in which the vector sum is written.

**Proof.** Let us prove the *commutative property of vector addition* in the case of two vectors $\vec{a}$ and $\vec{b}$.

Applying triangle law of vectors to the vector triangle ABC, we get

$$\vec{AC} = \vec{AB} + \vec{BC} \qquad \text{or} \qquad \vec{AC} = \vec{a} + \vec{b} \qquad \ldots(1)$$

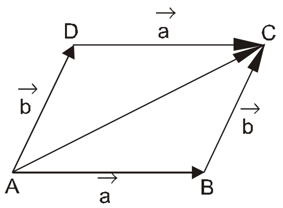



**Fig. 2.1.** Commutative property of vector addition

Again, applying triangle law of vectors to the vector triangle ADC, we get

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} \quad \text{or} \quad \overrightarrow{AC} = \vec{b} + \vec{a} \qquad \ldots(2)$$

From (1) and (2), $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

**(*iii*) Vector addition is distributive.**

According to distributive law of vector addition,

$$\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$$

**(*iv*) Vector addition is associative.** *The sum of the vectors remains the same in whatever grouping they are added.*

According to associative law of vector addition,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

**Proof.** Let the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be represented by $\overrightarrow{PQ}$, $\overrightarrow{QS}$ and $\overrightarrow{ST}$ respectively. There are two ways to calculate the resultant of these vectors.

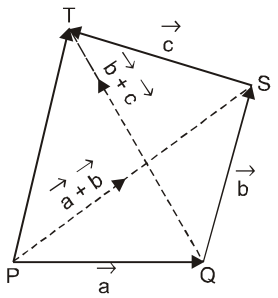



**Fig. 2.2.** Associative law of vector addition

The resultant of $\vec{a}$ and $\vec{b}$ is $\overrightarrow{PS}$ such that $\overrightarrow{PS} = \vec{a} + \vec{b}$

The resultant of $(\vec{a} + \vec{b})$ and $\vec{c}$ is $\overrightarrow{PT}$ such that

$$\overrightarrow{PT} = (\vec{a} + \vec{b}) + \vec{c} \qquad \ldots(1)$$

Again, if we add $\vec{b}$ and $\vec{c}$, we get $\overrightarrow{QT}$ such that $\overrightarrow{QT} = \vec{b} + \vec{c}$

The resultant of $\vec{a}$ and $(\vec{b} + \vec{c})$ is $\overrightarrow{PT}$ such that

$$\overrightarrow{PT} = \vec{a} + (\vec{b} + \vec{c}) \qquad \ldots(2)$$

From equations (1) and (2), $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ which proves the associative law of vector addition.

*In terms of order and grouping, the rules for vector addition are the same as those of scalar addition.*

## 2.2. RESOLUTION OF A VECTOR IN A PLANE

*Resolution of a vector is the process of splitting the vector into two or more vectors in different directions which together produce the same effect as is produced by the given vector.*

*The vectors into which the given vector is splitted are called **component vectors**.*

Consider two non-zero vectors $\vec{a}$ and $\vec{b}$ in a plane [Fig. 2.3]. Let $\vec{A}$ be any other vector in this plane. Through the tail (P) of $\vec{A}$, draw a straight line parallel to $\vec{a}$.

Similarly, draw a straight line, parallel to $\vec{b}$, through the terminal point (Q) of $\vec{A}$. Let both the lines intersect at C.

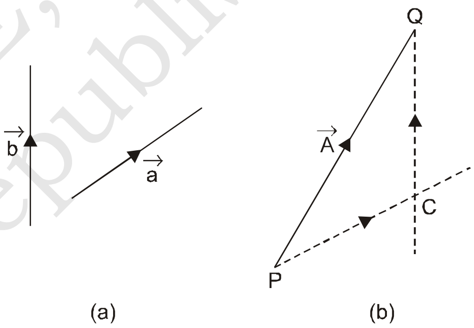



Fig. 2.3

Applying triangle law of vectors, $\vec{A} = \vec{PC} + \vec{CQ}$ ...(1)

As per the geometrical construction, $\vec{PC} = \lambda \vec{a}$ where $\lambda$ is a real number. In the given case, $\lambda$ is positive which indicates that $\vec{PC}$ is in the direction of $\vec{a}$. If $\lambda$ were negative, then $\vec{PC}$ would have been opposite to $\vec{a}$.

Similarly, $\vec{CQ} = \mu \vec{b}$, where $\mu$ is another real number.

From equation (1), $\vec{A} = \lambda \vec{a} + \mu \vec{b}$

So, $\vec{A}$ has been resolved along $\vec{a}$ and $\vec{b}$.

It may be noted that $\vec{A}$ determines μ and λ unambiguously. The converse is also true, *i.e.,* each vector $\vec{A}$ in a plane is completely described or determined by a pair of real numbers λ, μ. The **uniqueness of the resolution procedure** is proved below.

Let us assume that there are two ways of resolving $\vec{A}$ along $\vec{a}$ and $\vec{b}$ such that

$$\vec{a} = \lambda\,\vec{a} + \mu\,\vec{b} = \lambda'\,\vec{a} + \mu'\,\vec{b}$$

$$\therefore \qquad (\lambda - \lambda')\,\vec{a} = (\mu' - \mu)\,\vec{b}$$

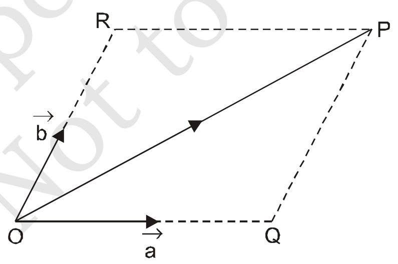
But $\vec{a}$ and $\vec{b}$ are different vectors.

So, the above equation is satisfied only if $\vec{a} = \vec{b} = \vec{0}$.

Thus, **there is one and only one way** in which a vector $\vec{A}$ can be resolved along $\vec{a}$ and $\vec{b}$. However, it may be pointed out here that a vector may be resolved into an infinite number of components. The reverse process, *i.e.,* the sum of the components will of course yield only the given vector.

In Fig. 2.4, the resolution of a position vector $\vec{OP}$ has been shown.

Applying parallelogram law of vectors, we can prove that $\lambda\,\vec{a}$ and $\mu\,\vec{b}$ are actually the components of $\vec{OP}$.




$$\vec{OQ} = \lambda\vec{a}, \quad \vec{OR} = \mu\vec{b}$$

**Fig. 2.4.** Resolution of vector

## 2.3. RECTANGULAR COMPONENTS

*When a vector is splitted into two component vectors at right angles to each other, the component vectors are called the* ***rectangular components*** *of the given vector.*

Consider a vector $\vec{A}$ represented by $\overrightarrow{OP}$. Through the point O, draw two mutually perpendicular axes—X-axis and Y-axis. Let the vector $\vec{A}$ make an angle θ with the X-axis. From the point P, drop a perpendicular PM on X-axis.

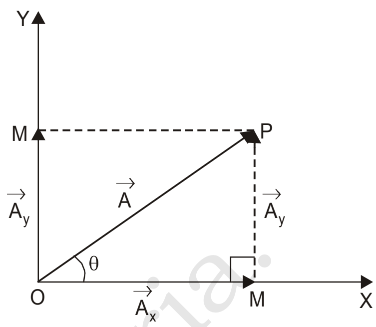



**Fig. 2.5.** Resolution of a vector into two rectangular components

Now $\overrightarrow{OM} (= \vec{A}_x)$ is the resolved part of $\vec{A}$ along X-axis. It is also known as the $x$-component of $\vec{A}$ or the horizontal component of $\vec{A}$. $\vec{A}_x$ may be regarded as the projection of $\vec{A}$ on X-axis.

$\overrightarrow{ON}$ (= $\vec{A}_y$ ) is the resolved part of $\vec{A}$ along Y-axis. It is also known as the $y$-component of $\vec{A}$ or the vertical component of $\vec{A}$. The vertical component of $\vec{A}$ may be regarded as the projection of $\vec{A}$ on Y-axis.

So, $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of $\vec{A}$.

Applying triangle law of vectors to the vector triangle OMP, we get

$$\vec{A}_x + \vec{A}_y = \vec{A}$$

This equation confirms that $\vec{A}_x$ and $\vec{A}_y$ are the components of $\vec{A}$.

In right-angled triangle OMP,

$$\cos\theta = \frac{A_x}{A} \quad \text{or} \quad A_x = A\cos\theta \qquad \ldots(1)$$

$$\sin\theta = \frac{A_y}{A} \quad \text{or} \quad A_y = A\sin\theta \qquad \ldots(2)$$

Squaring and adding (1) and (2), we get

$$A_x^2 + A_y^2 = A^2\cos^2\theta + A^2\sin^2\theta$$

or $$A_x^2 + A_y^2 = A^2(\cos^2\theta + \sin^2\theta)$$

∴ $$A_x^2 + A_y^2 = A^2 \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

or
$$\boxed{A = \sqrt{A_x^{\ 2} + A_y^{\ 2}}}$$

This equation gives the magnitude of the given vector in terms of the magnitudes of the components of the given vector.

## 2.4. RESOLUTION OF A POSITION VECTOR INTO TWO RECTANGULAR COMPONENTS

Fig. 2.6 shows position vector $\vec{r}$ represented by $\overrightarrow{OP}$. Draw PM ⊥ X-axis and PN ⊥ Y-axis. $\overrightarrow{OM} = x\,\hat{i}$ and $\overrightarrow{ON} = y\,\hat{j}$.

According to parallelogram law of vector addition,

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{ON}$$

or
$$\vec{r} = x\hat{i} + y\,\hat{j}$$

Let θ be the angle made by $\vec{r}$ with X-axis.

Then $x = r\cos\theta$ and $y = r\sin\theta$

$|\vec{r}|$ or $r = \sqrt{x^2 + y^2}$

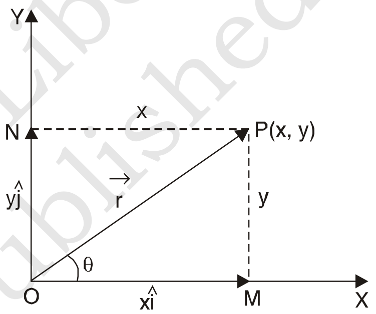


**Fig. 2.6.** Resolution of position vector

## 2.5. EXAMPLE OF RESOLUTION OF VECTOR

An example of 'resolution of a vector' is 'walk of a man'. When a man walks, he presses the ground slantingly in the backward direction with a force F. The ground offers an equal reaction R in the opposite direction. The vertical component V of this reaction balances the weight of the man. The horizontal component H helps the man to walk.

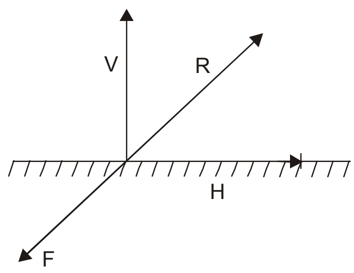


**Fig. 2.7.** Walk of a man

## 2.6. ADDITION OF VECTORS AND RECTANGULAR RESOLUTION

Consider two points P and Q having co-ordinates $(x_1, y_1)$ and $(x_2, y_2)$ respectively with reference to the origin O of the co-ordinate system. Let us first consider the position vector $\vec{r_1}$ which makes angle $\theta_1$ with X-axis.

Now, $$\vec{r_1} = \vec{x_1} + \vec{y_1} = x_1\hat{i} + y_1\hat{j}$$

$$x_1 = r_1 \cos\theta_1,\ y_1 = r_1 \sin\theta_1$$

$$r_1^2 = x_1^2 + y_1^2,\ \tan\theta_1 = \frac{y_1}{x_1}$$

Again, $(x_2 - x_1)$ and $(y_2 - y_1)$ are the components (in magnitude) of $\vec{PQ}$. Here, $y_2 - y_1$ is negative. [Note that $\vec{PQ}$ is directed from upper left to lower right.]

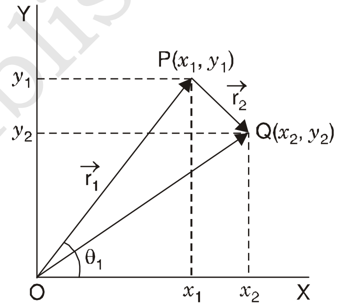


**Fig. 2.8**

Let us now add the components of $\vec{OP}$ to the components of $\vec{PQ}$.

Then, $x_1 + (x_2 - x_1) = x_2$, $y_1 + (y_2 - y_1) = y_2$

This gives us the components of $\vec{OQ}$.

So, we conclude that *the rule for addition of vectors can be broken down into two ordinary algebraic additions, one along each of the chosen axes. This directly implies that motion along a curve in a plane can be regarded as the sum of the independent linear motions, one along the X-axis and the other along Y-axis. The two linear motions may be treated separately and the results may be combined at the end.*

## 2.7. RESOLUTION OF A VECTOR INTO THREE RECTANGULAR COMPONENTS

Let a vector $\vec{A}$ be represented by $\vec{OP}$ as shown in Fig. 2.9. With O as origin, construct a rectangular parallelopiped with three edges along the three rectangular axes which meet at O. $\vec{A}$ becomes the diagonal of the parallelopiped. $\vec{A}_x$ , $\vec{A}_y$ and $\vec{A}_y$ are three vector intercepts along $x$, $y$ and $z$ axes respectively. These are the three rectangular components of $\vec{A}$ .

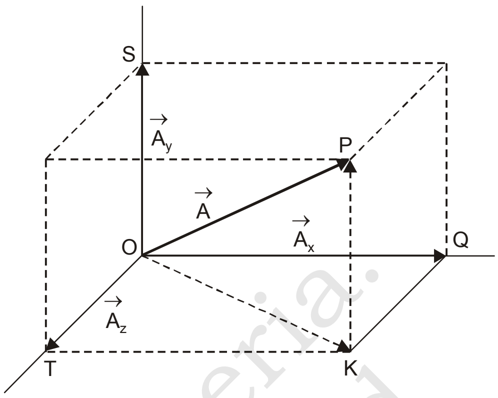



**Fig. 2.9.** Resolution of a vector into three rectangular components

Applying triangle law of vectors,

$$\vec{OP} = \vec{OK} + \vec{KP}$$

Applying parallelogram law of vectors, $\vec{OK} = \vec{OT} + \vec{OQ}$

$$\therefore \quad \vec{OP} = \vec{OT} + \vec{OQ} + \vec{KP}$$

But $\quad \vec{KP} = \vec{OS}$

$$\therefore \quad \vec{OP} = \vec{OT} + \vec{OQ} + \vec{OS}$$

$$\vec{A} = \vec{A}_z + \vec{A}_x + \vec{A}_y$$

or $$\boxed{\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z}$$

or $$\boxed{\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}}$$

Again, $\quad OP^2 = OK^2 + KP^2$

$$OP^2 = OQ^2 + QK^2 + KP^2$$

or $\quad OP^2 = OQ^2 + OT^2 + KP^2 \qquad [\because \quad QK = OT]$

or $\quad A^2 = A_x^2 + A_z^2 + A_y^2 \qquad [\because \quad KP = OS = A_y]$

or $\quad A^2 = A_x^2 + A_y^2 + A_z^2$

or $$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

This gives the magnitude of $\vec{A}$ in terms of the magnitudes of components $\vec{A}_x$ , $\vec{A}_y$ and $\vec{A}_z$ .

## 2.8. MOMENT OF FORCE (TORQUE)

**(*a*)** The rotational analogue of force is moment of force. It is also referred to as torque. This quantity measures the turning effect of a force.

*The* ***torque (or moment of force)*** *about an axis of rotation is a vector quantity, whose magnitude is equal to the product of magnitude of force and the perpendicular distance of the line of action of force from the axis of rotation and its direction is perpendicular to the plane containing the force and perpendicular distance.*

Fig. 2.10 shows a force $\vec{F}$ applied on a rigid body. The body is free to rotate about an axis passing through a point O and perpendicular to the plane of paper. If $d$ is the perpendicular distance of the line of action of force from the point O, then the torque $\tau$ about the axis of rotation is : $\tau = Fd$.

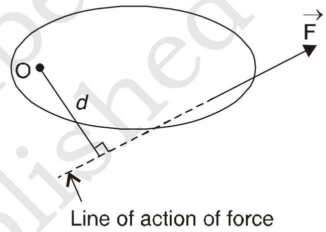


Fig. 2.10

The symbol $\tau$ stands for the Greek letter **tau.**

The torque is taken as positive if it tends to rotate the body anticlockwise. If the torque tends to rotate the body clockwise, then it is taken as negative.

The SI unit of torque is N m. Its dimensional formula is $[ML^2T^{-2}]$.

The dimensions of torque are the same as those of work or energy. It is, however, a very different physical quantity than work. Moment of force is a vector, while work is a scalar.

**(*b*) Torque in Vector Notation.** If a force $\vec{F}$ acts on a single particle at a point P whose position with respect to the origin O is given by the position vector $\vec{r}$, then the moment of force, acting on the particle, with respect to the origin O is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

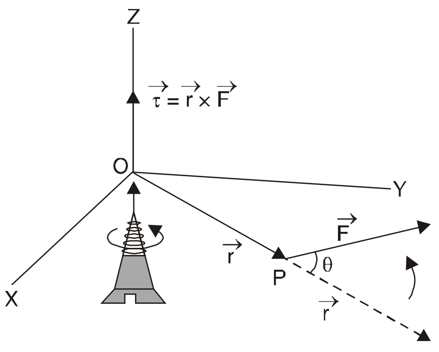


Fig. 2.11

The direction of $\vec{\tau}$ is perpendicular to the plane of $\vec{r}$ and $\vec{F}$. Its direction is given by right-handed screw rule or right-hand thumb rule.

The magnitude of $\vec{\tau}$ is given by $\tau = r\text{F} \sin\theta$ ...(1)

where $r$ is the magnitude of the position vector $\vec{r}$ *i.e.,* the length OP, F is the magnitude of the force $\vec{\text{F}}$ and $\theta$ is the angle between $\vec{r}$ and $\vec{\text{F}}$.

Now, $\sin\theta = \dfrac{d}{r}$ or $d = r \sin\theta$

From $= n(1),\ \tau = \text{F}\,(r \sin\theta) = \text{F}r_{\perp} = \text{F}d$

Again $\tau = r\,(\text{F} \sin\theta) = r\,\text{F}_{\perp} = r\,\text{F}_{\theta}$

## SPECIAL CASES

(*i*) If $r = 0$, then $\tau = 0$. Clearly, a force has no torque if it passes through the point O about which torque is to be calculated. *This explains as to why we cannot open or close a door by applying force at the hinges of the door.*

(*ii*) If $\theta = 0°$ or $180°$, then $\sin\theta = 0$.

$\therefore$ $t = r\,\text{F} \sin\theta =$
0

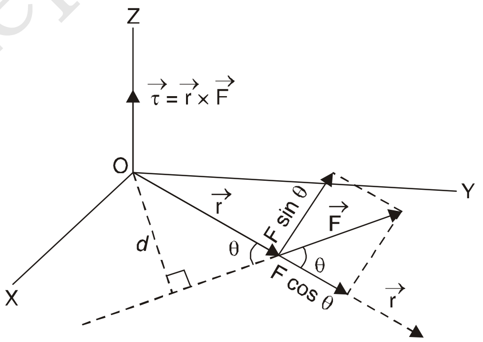


Fig. 2.12

In this case, the line of action of the force passes through point O. Thus, *if the line of action of force passes through point O, the torque is zero.*

(*iii*) If $\theta = 90°$, then $\sin\theta = \sin 90° =$ 1 (max. value). So, $\tau$ is maximum.

$$\tau_{\text{max.}} = r\text{F}$$

This explains as to why *a handle is fixed perpendicular to the plane of door.*



## 2.9. COUPLE

*A* ***couple*** *is a set of two equal (in magnitude), opposite (in direction) forces having different lines of action.* A couple produces rotation without translation.

**Properties of a Couple.** (*a*) A couple produces or tends to produce only the rotational motion. (*b*) A couple cannot be replaced by a single force. (*c*) A couple can be shifted anywhere in its plane of action.

Torque is the turning effect produced by a single force. Couple is only a set of two equal (in magnitude), opposite (in direction) and parallel forces having different lines of action.

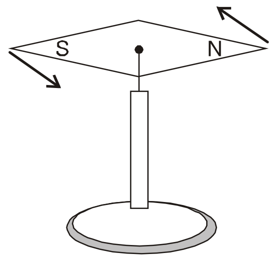



**Fig. 2.13.** The Earth's magnetic field exerts equal and opposite forces on the poles of a compass needle. These two forces form a couple

**Moment of Couple.** *It is the rotational effect produced by a couple.* It is a vector quantity. Its units and dimensions are the same as those of $\vec{\tau}$.

**Expression for Moment of Couple.** Let OX, OY and OZ be the three mutually perpendicular axes. Let two equal (in magnitude) and opposite (in direction) forces, $-\vec{F}$ and $\vec{F}$ act at P and Q respectively in the XOZ plane. The position vectors of P and Q with reference to origin O are given by $\vec{r}_1$ and $\vec{r}_2$ respectively.

Moment of force $-\vec{F}$ about O, $\vec{\tau}_1 = \vec{r}_1 \times (-\vec{F}) = -\vec{r}_1 \times \vec{F}$

Moment of force $\vec{F}$ about O, $\vec{\tau}_2 = \vec{r}_2 \times \vec{F}$

Applying the right-hand rule for the cross product of vectors, we find that $\vec{\tau}_1$ acts along the negative direction of Y-axis and $\vec{\tau}_2$ acts along the positive direction of Y-axis as shown in Fig. 2.14.

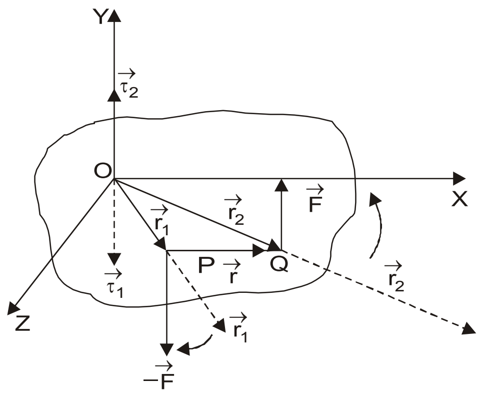



**Fig. 2.14.** Moment of couple

The moment of the couple $\vec{C}$ is *vector sum* of $\vec{r}_1$ and $\vec{r}_2$.

$$\therefore \quad \vec{C} = \vec{\tau}_1 + \vec{\tau}_2$$

$$= -\vec{r}_1 \times \vec{F} + \vec{r}_2 \times \vec{F} = \vec{r}_2 \times \vec{F} - \vec{r}_1 \times \vec{F}$$

or $$\vec{C} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$$

Applying *triangle law of vectors* to the vector triangle OPQ, we get

$$\vec{r_1} + \vec{r} = \vec{r_2} \qquad \text{or} \qquad \vec{r_2} - \vec{r_1} = \vec{r}$$

$$\therefore \qquad \boxed{\vec{C} = \vec{r} \times \vec{F}}$$

The vector $\vec{r}$ lies in the plane of the two forces, *i.e.*, the plane XOZ. $\vec{C}$ is perpendicular to this plane.

**Fig. 2.15.** Our fingers apply a couple to turn the lid

## 2.10. WORK DONE IN ROTATING A RIGID BODY

Consider a rigid body which is capable of rotation about an axis through a point O of the rigid body and perpendicular to the plane of the paper.

Consider a point P such that the position vector of P with respect to O is $\vec{r}$ [Fig. 2.16]. Suppose an external force $\vec{F}$ is applied at the point P as shown. Let the body turn through an infinitesimally small angle $d\theta$ in a short time $dt$ so that P moves to new position P′ such that $\vec{PP'} = \vec{ds}$.

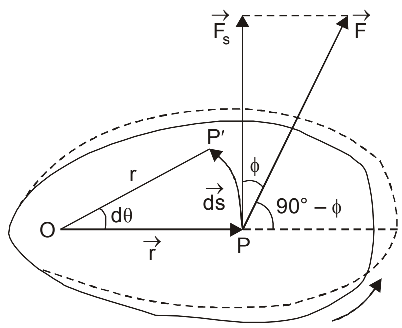



**Fig. 2.16.** Work done in rotational motion

*In magnitude, $ds = r\,d\theta$ ...(1)

Work $d$W done in rotating the body through a small angle $d\theta$ is given by

$$dW = \vec{F} \cdot \vec{ds} = F\,ds \cos \phi$$

* $\because \quad \theta = \frac{l}{r} \quad \therefore \quad l = r\theta$

Here, $l$ is the length of an arc of a circle of radius $r$, θ is the angle subtended by the arc at the centre of circle.

where $\phi$ is the angle between $\vec{F}$ and $\vec{ds}$.

Now, $$dW = (F \cos \phi)\, ds = F_s\, r\, d\theta \qquad \text{[From (1)]}$$

where $F_s$ (= F cos $\phi$) is the component of $\vec{F}$ in the direction of $\vec{ds}$. $\vec{F}_s$ is perpendicular to $\vec{r}$.

Again, $$dW = (F \cos \phi)\, r\, d\theta = (r F \cos \phi)\, d\theta$$

But $rF \cos \phi = rF \sin (90° - \phi) = |\vec{r} \times \vec{F}| = |\vec{\tau}| = \tau$

where $(90° - \phi)$ is the angle between $\vec{r}$ and $\vec{F}$. $\vec{\tau}$ is the moment of $\vec{F}$ about O.

$$\therefore \qquad dW = \vec{\tau} \cdot \vec{d\theta}$$

Both $\vec{\tau}$ and $\vec{d\theta}$ act in the same direction. So, the angle between them is 0°.

$$\therefore \qquad dW = \tau\, d\theta$$

$$\int dW = \int_0^\theta \tau\, d\theta \quad \text{or} \quad W = \tau \int_0^\theta d\theta = \tau(\theta - 0) = \tau\theta$$

or $$\boxed{W = \tau\, \theta}$$

Here it is assumed that $\tau$ is constant. Thus, the work done in rotating the body through a given angle is equal to the product of the torque and the angular displacement of the body.

## 2.11. POWER IN ROTATIONAL MOTION

Power, $$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta)$$

or $$P = \tau \frac{d}{dt}(\theta) \qquad \text{or} \qquad \boxed{P = \tau\, \omega}$$

*Note that $\tau$ is being assumed as a constant.*

## 2.12. EQUILIBRIUM OF RIGID BODIES

A **rigid body** is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time, or equivalently the body has neither linear acceleration nor angular acceleration.

A rigid body such as a chair, a bridge or building is said to be in equilibrium if both the linear momentum and the angular momentum of the rigid body have a constant value. When a rigid body is in equilibrium, the linear acceleration of its centre of mass is zero. Also, the angular acceleration of the rigid body about any fixed axis in the reference frame is zero.

For the equilibrium of a rigid body, it is not necessary that the rigid body is at rest. However, *if the rigid body is at rest, then the equilibrium of the rigid body is called* ***static equilibrium.***

**(*i*) First Condition for Equilibrium.** The translational motion of the centre of mass of a rigid body is governed by the following equation:

$$\Sigma \vec{F}_{ext.} = \frac{d}{dt}(\vec{p})$$

Here $\Sigma \vec{F}_{ext.}$ is the vector sum of all the external forces that act on the rigid body.

For equilibrium, $\vec{p}$ must have a constant value.

$$\therefore \quad \frac{d}{dt}(\vec{p}) = 0$$

$$\therefore \quad \Sigma \vec{F}_{ext.} = 0$$

This vector equation is equivalent to three scalar equations:

$$\sum_{i=1}^{n} F_{ix} = 0, \sum_{i=1}^{n} F_{iy} = 0, \sum_{i=1}^{n} F_{iz} = 0 \qquad ...(1)$$

This leads us to ***the first condition for the equilibrium of rigid bodies.***

***"The vector sum of all the external forces acting on the rigid body must be zero".***

**(*ii*) Second Condition for Equilibrium.** The rotational motion of a rigid body is governed by the following equation:

$$\Sigma \vec{\tau}_{ext.} = \frac{d\vec{L}}{dt}$$

Here $\Sigma \vec{\tau}_{ext.}$ represents the vector sum of all the external torques that act on the body.

For equilibrium, $\vec{L}$ must have a constant value.

$$\therefore \qquad \frac{d}{dt}(\vec{L}) = 0$$

$$\therefore \qquad \sum \vec{\tau}_{ext.} = 0$$

This vector equation can be written as three scalar equations:

$$\sum_{i=1}^{n} \tau_{ix} = 0,\ \sum_{i=1}^{n} \tau_{iy} = 0,\ \sum_{i=1}^{n} \tau_{iz} = 0 \qquad \ldots(2)$$

This leads us to ***the second condition for the equilibrium of rigid bodies.***

***"The vector sum of all the external torques acting on the rigid body must be zero."***

The second condition for equilibrium is independent of the choice of the origin and the co-ordinate axes used for calculating the components of torques. If the net torque is zero, then its components are zero for any choice of *x, y* and *z* axes.

A body may be in partial equilibrium *i.e.,* it may be in translational equilibrium and not in rotational equilibrium or it may be in rotational equilibrium and not in translational equilibrium.

Consider a light (*i.e.,* of negligible mass) rod (AB), at the two ends (A and B) of which two parallel forces both equal in magnitude are applied perpendicular to the rod as shown in Fig. 2.17.

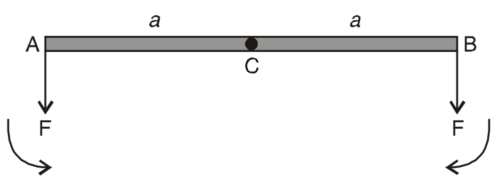



**Fig. 2.17**

Let C be the midpoint of AB. CA = CB = *a*. The moments of the forces at A and B, about C, will both be equal in magnitude (*a*F), but opposite in sense as shown. The net moment on the rod will be zero. The system will be in rotational equilibrium, but it will not be in translational equilibrium: $\Sigma F \neq 0$.

The force at B in Fig. 2.17 is reversed in Fig. 2.18. Thus, we have the same rod with two equal and opposite forces applied perpendicular to the rod, one at end A and the other at end B. Here the moments of both the forces are equal, but they are not opposite; they act in the same sense and cause anticlockwise rotation of the rod. The total force on the body is zero; so the body is in translational equilibrium; but it is not in rotational equilibrium. Although the rod is not fixed in any way, it undergoes pure rotation (*i.e.*, rotation without translation).

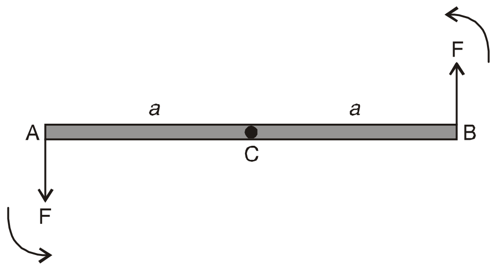


Fig. 2.18

## 2.13. PRINCIPLE OF MOMENTS (Case of Ideal Lever)

An ideal lever is essentially a light (*i.e.*, of negligible mass) rod pivoted at a point along its length. This point is called the fulcrum. A see-saw on the children's playground is a typical example of a lever. Two forces $F_1$ and $F_2$, parallel to each other and usually perpendicular to the lever, act on the lever at distances $d_1$ and $d_2$ respectively from the fulcrum as shown in Fig. 2.19.

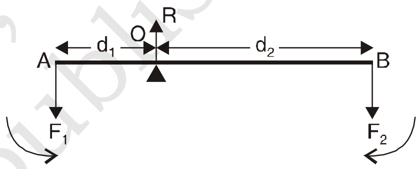


Fig. 2.19

Let R be the reaction of the support at the fulcrum. For translational equilibrium,

$$R - F_1 - F_2 = 0 \qquad ...(1)$$

For considering rotational equilibrium, we take the moments about the fulcrum ; the sum of moments must be zero.

$$F_1 d_1 - F_2 d_2 = 0 \qquad ...(2)$$

Normally the anticlockwise (clockwise) moments are taken to be positive (negative). Note R acts at the fulcrum itself and has zero moment about the fulcrum.

In the case of the lever, force $F_1$ is usually some weight to be lifted. It is called the *load* and its distance from the fulcrum $d_1$ is called the *load arm*. Force $F_2$ is the *effort* applied to lift the load ; distance $d_2$ of the effort from the fulcrum is the *effort arm*.

Eq. (2) can be written as

$$F_1 d_1 = F_2 d_2$$

or load × load arm = effort × effort arm

The above equation expresses the principle of moments for a lever. Incidentally the ratio $F_1/F_2$ is called the Mechanical Advantage (M.A.);

$$\text{M.A.} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

If the effort arm $d_2$ is larger than the load arm, the mechanical advantage is greater than one. Mechanical advantage greater than one means that a small effort can be used to lift a large load.

## 2.14. CENTRE OF GRAVITY

*The centre of gravity of a body is a point where the weight of the body acts and total gravitational torque on the body is zero.*

Consider an irregular-shaped cardboard and a narrow tipped object like a pencil. By trial and error, we can locate a point G on the cardboard where it can be balanced on the tip of the pencil. (The cardboard remains horizontal in this position.) This point of balance is the centre of gravity (CG) of the cardboard. The tip of the pencil provides a vertically upward force due to which the cardboard is in mechanical equilibrium. As shown in Fig. 2.20, the reaction of the tip is equal and opposite to $\overrightarrow{Mg}$, the total weight of (*i.e.,* the force of gravity on) the cardboard and hence the cardboard is in translational equilibrium. It is also in rotational equilibrium; if it were not so, due to the unbalanced torque it would tilt and fall. There are torques on the cardboard due to the forces of gravity like $\overrightarrow{m_1 g}$, $\overrightarrow{m_2 g}$ ...... etc., acting on the individual particles that make up the cardboard.

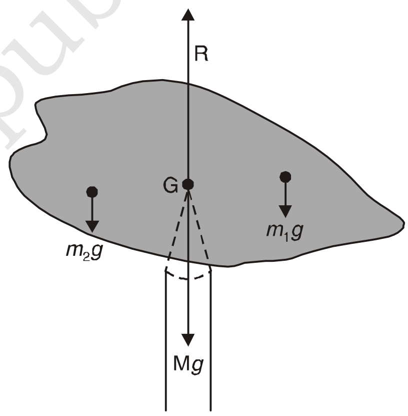


**Fig. 2.20.** Balancing a cardboard on the tip of a pencil. The point of support G is the centre of gravity

The CG of the cardboard is so located that the total torque on it due to the forces $\overrightarrow{m_1 g}$, $\overrightarrow{m_2 g}$ ...... etc. is zero.

If $\vec{r_i}$ is the position vector of the *i*th particle of an extended body with respect to its CG, then the torque about the CG, due to the force of gravity on the particle is $\vec{\tau_i} = \vec{r_i} \times \overrightarrow{m_i g}$. The total gravitational torque about the CG is zero, *i.e.*, $\vec{\tau_g} = \Sigma \vec{\tau_i} = \Sigma \vec{r_i} \times \overrightarrow{m_i g} = 0$ ...(1)

We may therefore, define the CG of a body as that point where the total gravitational torque on the body is zero.

In Eq. (1), $\vec{g}$ is the same for all particles, and hence it comes out of the summation. This gives, since $\vec{g}$ is non-zero,

$\Sigma m_i \vec{r_i} = 0$. The position vectors ( $\vec{r_i}$ ) are taken with respect to the CG. So, the origin must be the centre of mass of the body. Thus, the centre of gravity of the body coincides with the centre of mass in uniform gravity or gravity-free space.

## 2.15. STABLE, UNSTABLE AND NEUTRAL EQUILIBRIUM OF RIGID BODIES

Something is in equilibrium when both the resultant force and resultant turning moment on it are zero.

Following are the three types of translational static equilibrium of a body.

(*i*) ***When potential energy is minimum, the particle is said to be in stable equilibrium.***

Any displacement of the particle from the equilibrium position will result in a restoring force. This restoring force shall try to return the particle to the equilibrium position.

If a body is in stable equilibrium, work must be done on it by an external agent to change its position. This results in an increase in its potential energy.

Consider a cube at rest on one face on a horizontal table. Fig. 2.21 shows the central cross-section of the cube. The centre of gravity is shown at the centre of this cross-section. Suppose a force F is applied to the cube so as to rotate it without slipping about an axis along an edge. The centre of gravity of the cube will be raised. Moreover, work is done on the cube. This increases the potential energy of the cube. If the force is removed, the cube tends to return to its original position. This initial position is clearly a stable equilibrium position.

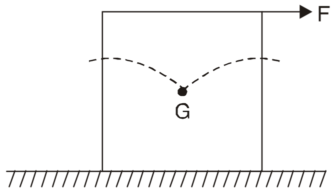



**Fig. 2.21.** Stable equilibrium

(*ii*) ***When the potential energy of a system is maximum, the system is in unstable equilibrium.***

Any displacement from 'unstable equilibrium position' will result in a force tending to push the system farther from the 'unstable equilibrium position'. No work is required to be done on the system by an external agent to change the position of the system. The displacement results in a decrease in the potential energy of the system.

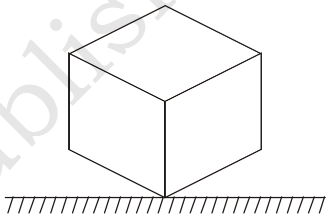


**Fig. 2.22.** Unstable equilibrium

A cube balanced on an edge can be considered in unstable equilibrium if a horizontal force is applied perpendicular to the edge. But the cube is in stable equilibrium with respect to a horizontal force parallel to the edge.

(*iii*) ***When the potential energy of a system is constant, the system is said to be in neutral equilibrium.***

When the system is displaced slightly, there is neither a repelling nor a restoring force.

A sphere (say, a football) on a horizontal table is a good illustration of neutral equilibrium. If a horizontal force is applied on the sphere, the centre of gravity of the sphere is neither raised nor lowered. The centre of gravity moves along the dashed line in Fig. 2.23. The potential energy of the sphere remains constant during displacement.

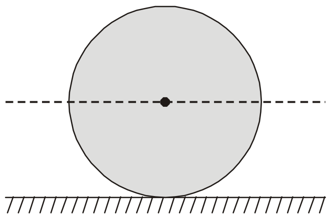


**Fig. 2.23.** Neutral equilibrium

## 2.16. CONCEPT OF CENTRE OF MASS

*The centre of mass of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translatory motion.*

*The centre of mass of a system of particles is that single point which moves in the same way in which a single particle having the total mass of the system and acted upon by the same external force would move.*

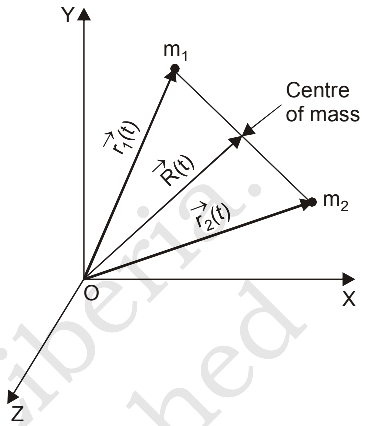



**Fig. 2.24.** Centre of mass of a two-particle system

The centre of mass of a system is only a point defined mathematically for the sake of convenience. It is not necessary that the total mass of the system be actually present at the centre of mass. As an example, centre of mass of a uniform circular ring is at the centre of the ring where there is no mass.

It may be noted that *it is not necessary that there may be a material particle at the centre of mass of the system.* But we can always calculate the position of the centre of mass at each time.

(*i*) For a two-particle system, the centre of mass always lies between the two particles and on the line joining them. In-fact $\vec{R}$ is a weighted average *i.e.,* each particle makes a contribution proportional to its mass.

(*ii*) When the two particles are of equal masses *i.e.*, $m_1 = m_2 = m$(say), then

$$\vec{R} = \frac{m\vec{r}_2 + m\vec{r}_2}{m+m} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

So, the centre of mass of two particles of equal masses lies exactly midway between them.

## 2.17. FRICTION

***Friction*** *is the retarding force which is called into play when a body actually moves or tends to move over the surface of another body.*

Consider a block of mass $m$ which is projected with initial velocity $v$ along a long horizontal table. The block will finally come to rest. This means that while it is moving, it experiences an opposing force that points in a direction opposite to its motion. This opposing force is called force of friction.

Whenever the surface of one body slides over that of another, each body exerts a frictional force on the other. The frictional force on each body is in a direction opposite to its motion relative to the other body. *Frictional forces always oppose relative motion and never help it.* **Even when no relative motion is actually present but there is only a tendency for relative motion, frictional forces exist between surfaces.**

Friction is very important in our daily lives. Left to act alone, it brings every moving body to a stop. In an automobile, nearly 20% of the engine power is used to counteract frictional forces. Friction causes wear and tear of the moving parts and many engineering man-hours are devoted to reducing it. On the other hand, without friction, we could not walk, we could not hold a pen and if could it would not write; wheeled transport as we know it would not be possible.

Consider a block at rest on a horizontal table. We find that the block will not move even though we apply a small force [Fig. 2.25(1)]. The applied force is clearly balanced by an opposite frictional force exerted on the block by the table, acting along the surface of contact. As the applied force is gradually increased, the frictional force $f_s$ also increases [Figs. 2.25(2) and 2.25(3)]. This indicates the **self-adjusting nature of the frictional force.**

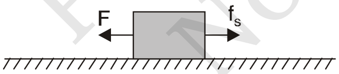


Fig. 2.25 (1)

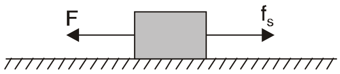


Fig. 2.25 (2)

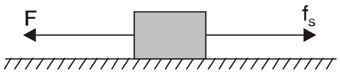


Fig. 2.25 (3)

*The frictional forces acting between surfaces at rest with respect to each other are called **forces of static friction.***

As we continue to increase the applied force, *we find some definite force at which the block just begins to move* [Fig. 2.25(4)]. At this stage, the maximum force of static friction acts. *The maximum force of static friction will be the same as the smallest force necessary to start motion.*

Once motion is started, the frictional force decreases so that a smaller force is necessary to maintain uniform motion [Fig. 2.25 (5)]. The forces acting between surfaces in relative motion are called forces of kinetic friction.

If the applied force is greater than the force of kinetic friction, then the block has accelerated motion [Fig. 2.25 (6)].

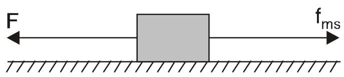



Fig. 2.25 (4)

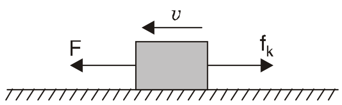



Fig. 2.25 (5)

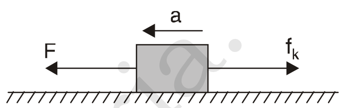



Fig. 2.25 (6)

## 2.18. STATIC FRICTION

*It is the force of friction which exactly balances the applied force during the stationary state of the body.* This frictional force exists when the bodies in contact are at rest with respect to each other. The force of static friction is a self-adjusting force *i.e.,* it adjusts its magnitude and direction so as to become exactly equal and opposite to the applied pull. The direction of the force of friction remains always opposite to the direction of the applied force.

Consider a block resting on a horizontal surface [Fig. 2.26]. Let a small pull P be applied on the body as shown. Let $f_s$ be the resulting force of static friction. In the equilibrium position, the weight W of the body will be balanced by the normal reaction R. And the applied pull P will be balanced by the frictional force $f_s$.

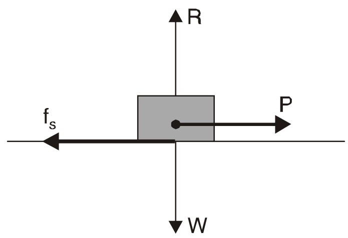



**Fig. 2.26.** Static friction

In vector notation, $\vec{W} = -\vec{R}$

and $\vec{P} = -\vec{f_s}$

## 2.19. LIMITING FRICTION

***Limiting friction*** *is the maximum value of static friction which is called into play when a body is just going to start sliding over the surface of another body.*

When the applied pull P is increased, the static frictional force $f_s$ also increases. However, there is a particular limit upto which the static frictional force $f_s$ can increase. Beyond this limit, the applied pull P will be able to produce motion in the body.

## 2.20. LAWS OF LIMITING FRICTION

Following are the laws of limiting friction:

**I.** *The direction of the force of limiting friction is always opposite to that in which the motion tends to take place.*

**II.** *The limiting friction acts tangentially to the two surfaces in contact.*

**III.** *The magnitude of the limiting friction is directly proportional to the normal reaction between the two surfaces.*

**IV.** *The limiting friction depends upon the material and the nature of the surfaces in contact and their state of polish.*

**V.** *For any two given surfaces, the magnitude of the limiting friction is independent of the shape or the area of the surfaces in contact so long as the normal reaction remains the same.*

**Experimental Verification.** Consider a wooden block placed on a horizontal surface. It is attached to a string which passes over a frictionless pulley carrying a scale pan at the free end [Fig. 2.27]. Add weights in the scale pan till wooden block just starts sliding. It is evident that the force of friction was opposing the motion. This verifies law I.

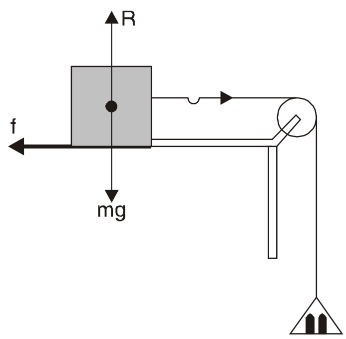


**Fig. 2.27.** Experimental verification of the laws of limiting friction

The force of friction $f$ acts along the horizontal surface. This verifies law II.

When the block just starts sliding, the total weight added in the scale pan along with the weight of the scale pan is equal to the limiting friction. The normal reaction R is equal to the weight $mg$ of the block. Now, put some known weight on the block. Determine the limiting friction again. It will be observed that the ratio of limiting friction and normal reaction is constant. In other words, the limiting friction is proportional to the normal reaction R. This verifies law III.

Let the wooden block be replaced by glass block of the same weight *mg*. It will be observed that the limiting friction will be different in this case. This verifies law IV.

If the wooden block is placed on its side instead of on its base, it will be observed that same force is required to move the block as when it was placed on its base. This verifies law V.

## 2.21. DYNAMIC OR KINETIC FRICTION

***Dynamic*** *or* ***kinetic friction*** *comes into play if the two bodies in contact are in relative motion.* It acts in a direction opposite to the direction of the instantaneous velocity.

The dynamic or kinetic friction is of the following two types :

**(*i*) Sliding friction.** *It comes into play when a solid body slides over the surface of another body.*

**(*ii*) Rolling friction.** *It comes into play when a body rolls over the surface of another body.*

## 2.22. LAWS OF SLIDING FRICTION

**(*i*)** The sliding friction opposes the applied force and has a constant value, depending upon the nature of the two surfaces in relative motion.

**(*ii*)** The force of sliding friction is directly proportional to the normal reaction R.

**(*iii*)** The sliding frictional force is independent of the area of the contact between the two surfaces so long as the normal reaction remains the same.

**(*iv*)** The sliding friction does not depend upon the velocity, provided the velocity is neither too large nor too small.

## 2.23. VARIATION OF FRICTIONAL FORCE WITH THE APPLIED FORCE

It is illustrated graphically in Fig. 2.28. When there is no relative motion between the two bodies in contact, the frictional force increases at the same rate as the applied force.

If ON′ is the applied force, then ON is the frictional force such that

$$ON' = ON$$

The slope of the curve O*a* is constant and is equal to unity.

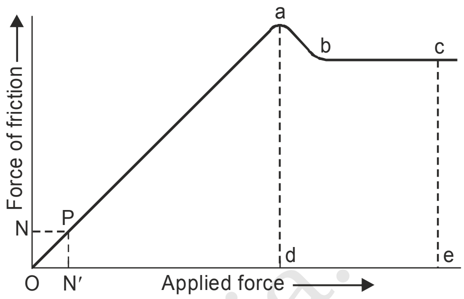



**Fig. 2.28.** Variation of frictional force with the applied force

When the applied force is equal to O*d*, the static frictional force becomes maximum. So, *ad* represents the limiting friction. When the applied pull exceeds the value O*d*, the body begins to move. At this stage, the frictional force suddenly decreases by a small amount and acquires a constant value *ce.* This value represents the dynamic or kinetic or sliding frictional force.

## 2.24. COEFFICIENT OF STATIC FRICTION

*For any two surfaces in contact, it is the ratio of the limiting friction $f_{ms}$ and the normal reaction R between them.* It is denoted by $\mu_s$.

$$\mu_s = \frac{f_{ms}}{R}$$

Since $\mu_s$ is a pure ratio therefore it has no units. The value of $\mu_s$ depends upon the state of polish of the two surfaces in contact. If the surfaces are smooth, the value of $\mu_s$ is small.

The force of static friction $f_s$ is equal to the applied force. So, $f_s$ can have any value from 0 to $f_{ms}$.

$$\therefore \qquad f_s \le f_{ms}$$

[The equality sign holds only when $f_s$ has its maximum value.]

$$\therefore \qquad f_s \le \mu_s R$$

## 2.25. COEFFICIENT OF KINETIC FRICTION

*It is defined as the ratio of kinetic friction and normal reaction.* It is denoted by $\mu_k$.

$\therefore \qquad \mu_k = \dfrac{f_k}{R}$

Now, $\qquad \dfrac{\mu_s}{\mu_k} = \dfrac{f_{ms}}{R} \times \dfrac{R}{f_k} = \dfrac{f_{ms}}{f_k}$

But $\qquad f_{ms} > f_k$

$\therefore \qquad \boxed{\mu_s > \mu_k}$

## 2.26. ANGLE OF FRICTION

*It is the angle which the resultant of the force of limiting friction* $\vec{f}_{ms}$ *and the normal reaction* $\vec{R}$ *makes with the normal reaction* $\vec{R}$.

Consider a block of weight $\vec{W}$ resting on a horizontal surface. The weight $\vec{W}$ will be balanced by the normal reaction $\vec{R}$ [Fig. 2.29].

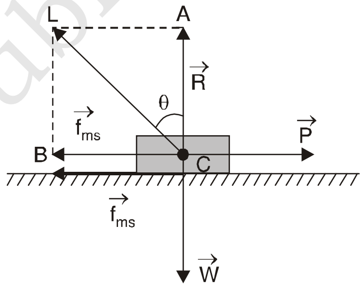


**Fig. 2.29.** Angle of friction

In vector notation, $\vec{W} = -\vec{R}$ (Newton's 3rd law of motion)

Now, apply a horizontal force $\vec{P}$ of such a magnitude that the block is about to move. Then, CB will represent the maximum force of static friction *i.e.,* limiting friction. The resultant of the limiting friction and the normal reaction is represented by the diagonal CL of the parallelogram CBLA. The angle θ which the resultant makes with the normal reaction is called the angle of friction.

In ΔCAL, $\qquad \tan\theta = \dfrac{AL}{CA} = \dfrac{CB}{CA} = \dfrac{f_{ms}}{R}$

But $\qquad \dfrac{f_{ms}}{R} = \mu_s \qquad$ (definition of coefficient of friction)

$\therefore \qquad \boxed{\tan\theta = \mu_s}$

So, *the tangent of the angle of friction is equal to the coefficient of static friction.*

## 2.27. ROLLING FRICTION

When a body rolls or tends to roll over the surface of another body, then both the rolling body and the surface on which it rolls are compressed by a small amount. As a result, the rolling body has to continuously climb a hill as shown [Fig. 2.30]. Apart from this, the rolling body has to continuously detach itself from the surface on which it rolls. This is opposed by the adhesive force between the two surfaces in contact. On account of both these factors, a force originates which retards the rolling motion. This retarding force is called the rolling friction. It is denoted by $f_r$.

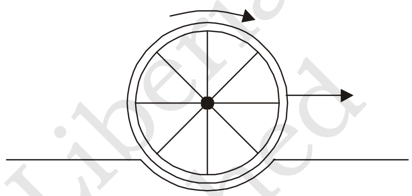


**Fig. 2.30.** Cause of rolling friction

**Laws of rolling friction.** The following laws of rolling friction are based on experiments.

***(i)*** Rolling friction is directly proportional to normal reaction.

$$f_r \propto \mathrm{R}$$

***(ii)*** Rolling friction is inversely proportional to the radius of the rolling body.

$$f_r \propto \frac{1}{r}$$

Combining the two laws, we get

$$f_r \propto \frac{\mathrm{R}}{r}$$

or $$\boxed{f_r = \mu_r \times \frac{\mathrm{R}}{r}} \qquad \ldots(1)$$

where $\mu_r$ is the coefficient of rolling friction, R is the normal reaction and $r$ is the radius of the rolling body.

**Comparison.** For the same magnitude of normal reaction, the sliding friction is much greater than the rolling friction. That is why we prefer to convert sliding friction into rolling friction. The ball and roller bearings make use of this principle.

**Illustration.** The sliding friction of steel on steel is 100 times more than the rolling friction of steel on steel.

## 2.28. FRICTION IS A NECESSARY EVIL

### Friction is a Necessity

(*i*) Without friction between our feet and the ground, it will not be possible to walk. When the ground becomes slippery after rain, it is made rough by spreading sand, etc.

(*ii*) The tyres of the vehicles are made rough to increase friction.

(*iii*) Various parts of a machine are able to rotate due to friction between belt and pulley.

### Friction is an Evil

(*i*) Wear and tear of the machinery is due to friction.

(*ii*) Friction between different parts of the rotating machines produces heat and causes damage to them.

(*iii*) We have to apply extra power to machines in order to overcome friction. Thus, the efficiency of the machines decreases.

## 2.29. METHODS OF REDUCING FRICTION

(*i*) **Polishing.** The interlocking and the projections between the two surfaces are minimised and therefore the friction is reduced. This makes their life long.

(*ii*) **Lubrication.** A lubricant is a substance (a solid or a liquid) which forms thin layer between the two surfaces in contact. It fills the depressions present in the surfaces of contact and hence friction is reduced.

(*iii*) **Streamlining.** When a body moves past a fluid (liquid or air), the particles of the fluid move past it in regular lines of flow called streamlines. It is found that the resistance offered by the fluid to the body is minimum when its shape resembles that of streamlines. Thus the shape of automobiles is so designed that it resembles the streamline pattern and the resistance offered by the fluid is minimum.

***(iv)* Avoiding moisture.** When the moisture is present, the friction is more. So, we must avoid moisture between the two surfaces.

***(v)* Use of alloys.** Friction is reduced by lining the moving parts with alloys because alloys have low coefficients of friction.

***(vi)* Use of ball-bearings or roller-bearings.** The rolling friction is much less than the sliding friction. So, we convert sliding friction into rolling friction. Even the axle is not allowed to move directly in the hub. The friction is further minimised by the use of roller-bearings or ball-bearings [Fig. 2.31].

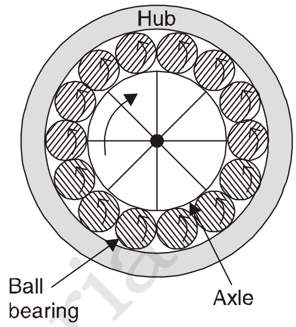



**Fig. 2.31.** Ball-bearings

## 2.30. WHY IS IT EASIER TO PULL A BODY THAN TO PUSH IT ?

Suppose a force P is applied to pull a block of weight W [Fig. 2.32]. The force P can be resolved into two rectangular components : P cos $\theta$ and P sin $\theta$.

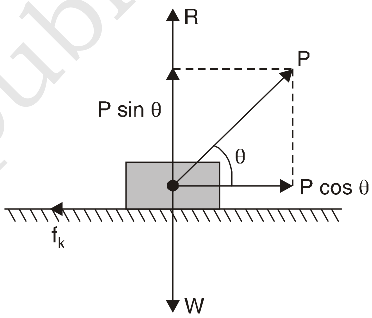



**Fig. 2.32.** Pulling a block

If R be the normal reaction, then

$$R = W - P \sin \theta$$

Force of kinetic friction,

$$f_k = \mu_k R$$

or $$f_k = \mu_k (W - P \sin \theta) \qquad ...(1)$$

If a force P is applied to push a block of weight W [Fig. 2.33], then normal reaction,

$$R' = W + P \sin \theta$$

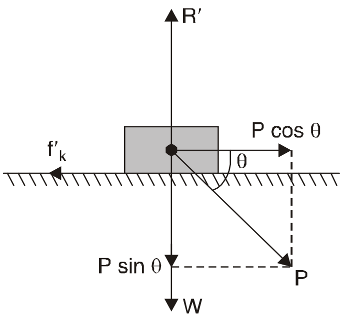



**Fig. 2.33.** Pushing a block

force of kinetic friction, $f_k' = \mu_k R'$

or $$f_k' = \mu_k (W + P \sin \theta) \qquad ...(2)$$

Comparing (1) and (2), we find that

$$f_k' > f_k$$

So, the frictional force is more in the case of push.

Hence, it is easier to pull than to push a body.

## REVIEW EXERCISES

**Do the review exercises in your notebook.**

### A. Multiple Choice Questions

**1.** A circular disk of radius R is made from an iron plate of thickness $t$ and another disc Y of radius 4R is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moments of inertia $I_X$ and $I_Y$ is

(*a*) $I_Y = I_X$ (*b*) $I_Y = 64\ I_X$

(*c*) $I_Y = 32\ I_X$ (*d*) $I_Y = 16\ I_X$.

**2.** A thin circular ring of mass M and radius $r$ is rotating about its axis with a constant angular velocity ω. Four objects, each of mass $m$, are kept gently on the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(*a*) $\frac{M\omega}{4m}$ (*b*) $\frac{M\omega}{4m}$

(*c*) $\frac{(M+4m)\,\omega}{M}$ (*d*) $\frac{(M-4m)\,\omega}{M+4m}$.

**3.** One end of a thin uniform rod of length L and mass $M_1$ is riveted to the centre of a uniform circular disc of radius $r$ and mass $M_2$ so that both are coplanar. The centre of mass of the combination from the centre of the disc is (assume that the point of attachment is at the origin)

(*a*) $\frac{L(M_1+M_2)}{2M_1}$ (*b*) $\frac{LM_1}{2(M_1+M_2)}$

(*c*) $\frac{2(M_1+M_2)}{LM_1}$ (*d*) $\frac{2LM_1}{(M_1+M_2)}$.

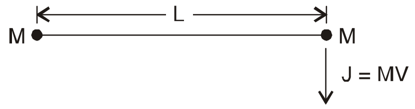
**4.** Two circular loops A and B of radii $r_A$ and $r_B$ respectively are made from the same uniform wire. The ratio of their moments of inertia about axes passing through their centres and perpendicular to their planes is $I_B/I_A = 8$. Then $(r_B/r_A) =$

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8.

**5.** Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse

J = MV is imparted to the body at one of its ends, what would be its angular velocity?




(*a*) V/L (*b*) 2V/L

(*c*) V/3L (*d*) V/4L.

**6.** A turntable rotates about a vertical axis with a constant angular speed $\omega$. A circular pan rests on the turntable and rotates along with the table. The bottom of the pan is covered with a uniform thick layer of ice which also rotates with the pan. The ice starts melting. The angular speed of the turntable

(*a*) decreases (*b*) increases

(*c*) remains the same as $\omega$ (*d*) data insufficient.

**7.** Water is poured from a height of 10 m into an empty barrel at the rate of 1 litre per second. If the weight of the barrel is 10 kg, the weight indicated at time $t$ = 60 s will be

(*a*) 71.4 kg (*b*) 68.6 kg

(*c*) 70.0 kg (*d*) 84.0 kg.

**8.** A force of 200 N is required to push a car of mass 500 kg slowly at constant speed on a level road. If a force of 500 N is applied, the acceleration of the car (in m $s^{-2}$) will be

(*a*) zero (*b*) 0.2

(*c*) 0.6 (*d*) 1.0.

**9.** When a bucket containing water is rotated fast in a vertical circle of radius R, the water in the bucket doesn't spill provided

(*a*) The bucket is whirled with a maximum speed of $\sqrt{2gR}$.

(*b*) The bucket is whirled around with a minimum speed of $\sqrt{\frac{gR}{2}}$.

(*c*) The bucket is having a r.p.m. of $n = \sqrt{\frac{900\,g}{\pi^2 R}}$.

(*d*) The bucket is having a r.p.m. of $n = \sqrt{\frac{3600\,g}{\pi^2 R}}$.

**10.** An insect is crawling up on the concave surface of a fixed hemispherical bowl of radius R. If the coefficient of friction is $\frac{1}{3}$, then the height up to which the insect can crawl is nearly

(*a*) 5% of R (*b*) 6% of R

(*c*) 6.5% of R (*d*) 7.5% of R.

### B. Fill in the Blanks

1. A mass of 1 kg is just able to slide down the slope of an inclined rough surface when the angle of inclination is 60°. The minimum force necessary to pull the mass up the inclined plane is ($g = 10$ m $s^{-2}$) is __________ .
2. A block of mass m is resting on a smooth horizontal surface. One end of a uniform rope of mass (m/3) is fixed to the block, which is pulled in the horizontal direction by applying a force F at the other end. The tension in the middle of the rope is __________ .
3. A motor car is moving with a speed of 20 m $s^{-1}$ on a circular track of radius 100 m. If its speed is increasing at the rate of 3 m $s^{-1}$, its resultant acceleration is __________ .
4. A body of mass 0.05 kg is observed to fall with an acceleration of 9.5 m $s^{-2}$. The opposing force of air on the body is __________ ($g = 9.8$ m $s^{-2}$).
5. A car of mass 1500 kg is moving with a speed of 12.5 m $s^{-1}$ on a circular path of radius 20 m on a level road. The value of coefficient of friction between the tyres and road, so that the car does not slip, is __________ .

### C. Very Short Answer Questions

1. Is it possible that a particle moving with constant speed may not have a constant velocity? If yes, give an example.
2. A stone is rotated in a circle with a string. The string suddenly breaks. In which direction will the stone move?
3. What is the source of centripetal force in the case of an electron revolving around the nucleus?
4. What is the effect on the direction of the centripetal force when the revolving body reverses its direction of motion?
5. Is it correct to say that the banking of roads reduces the wear and tear of the tyres of automobiles? If yes, explain.

### D. Short Answer Questions

1. A stone tied to the end of a string is whirled in a horizotnal circle. When the string breaks, the stone flies away tangentially. Why?
2. What is the acceleration of a train travelling at 40 m $s^{-1}$ as it goes round a curve of 160 m radius?
3. Is the angular velocity of rotation of hour hand of a watch greater or smaller than the angular velocity of Earth's rotation about its own axis?

4. (i) What is the direction of the angular velocity of the minute hand of a wall-clock?
   (ii) When the car takes a turn round a curve, a passenger sitting in the car tends to slide. To which side does the passenger slide?
   (iii) Comment on the statement 'sharper the curve, more is the bending'.
5. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

## E. Long Answer Questions

1. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.
2. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.
3. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?
4. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad $s^{-1}$. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of the angular momentum of the cylinder about its axis?
5. A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rpm. How much is the angular speed of revolution of the child if he folds his hands back and thereby reduces his moment of inertia to $\frac{2}{5}$ times the initial value? Assume that the turntable rotates without friction.